Orbits in Space and Time
the strange effects of General Relativity

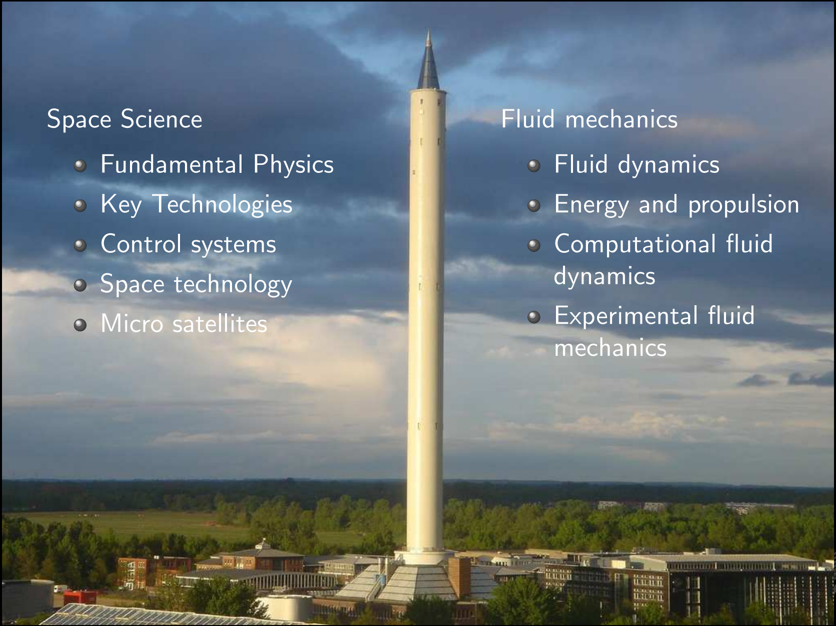
Claus Lämmerzahl

Center for Applied Space Technology and Microgravity (ZARM)

University of Bremen

Astronomical Institute
Tashkent, 7. March 2011

The Bremen drop tower



Space Science

- Fundamental Physics
- Key Technologies
- Control systems
- Space technology
- Micro satellites

Fluid mechanics

- Fluid dynamics
- Energy and propulsion
- Computational fluid dynamics
- Experimental fluid mechanics

Fundamental Physics at ZARM: Scope

Scope of Fundamental Physics at ZARM

- Development of new technologies
  - for microgravity experiments (drop tower, ISS, satellite)
  - for applications in space
- Accompanying theoretical investigations
  - motivation for experiments and missions
  - theory for experiments and applications
- High precision modeling
  - experimental devices
  - whole spacecraft
  - whole missions
  - quantum modeling

23 members

3 Professors, 3 post–docs, 14 PhD students, 1 diploma student, 2 technicians

Fundamental Physics at ZARM

Center of Applied Space Technology and Microgravity

Research areas

Fundamental Physics at ZARM

Center of Applied Space Technology and Microgravity

Research areas

- Satellite dynamics
  - modeling
  - disturbance forces
  - thermal and stress analysis
  - HPS (High Performance satellite dynamics Simulator)

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  - atom interferometry (exp & theory)
  - quantum tests (equivalence principle, decoherence, linearity, ...)
  - development of corresponding space technology

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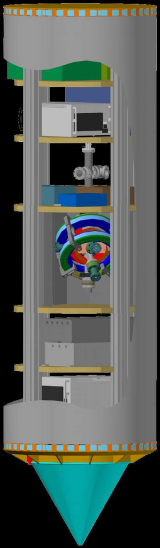
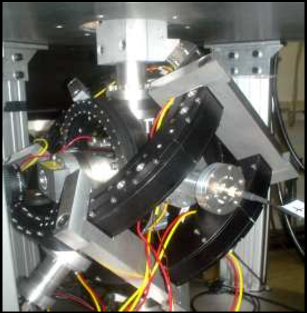
  - development of corresponding space technology
- Gravitational physics
  - tests of Equivalence Principle
  - analytical and numerical solutions for orbits
  - application to space
  - quantum gravity phenomenology
  - theoretical description of experiments testing SR and GR

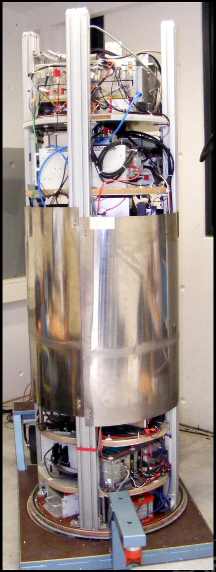
BEC in microgravity



design of capsule



vacuum chamber



capsule

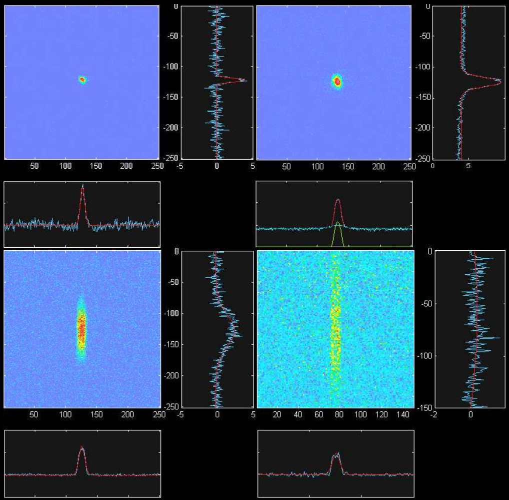
First BEC in microgravity / extended free fall

LU Hannover, ZARM, MPQ Munich, U Hamburg, HU Berlin, U Ulm

BEC in microgravity – long free evolution

50 ms

100 ms



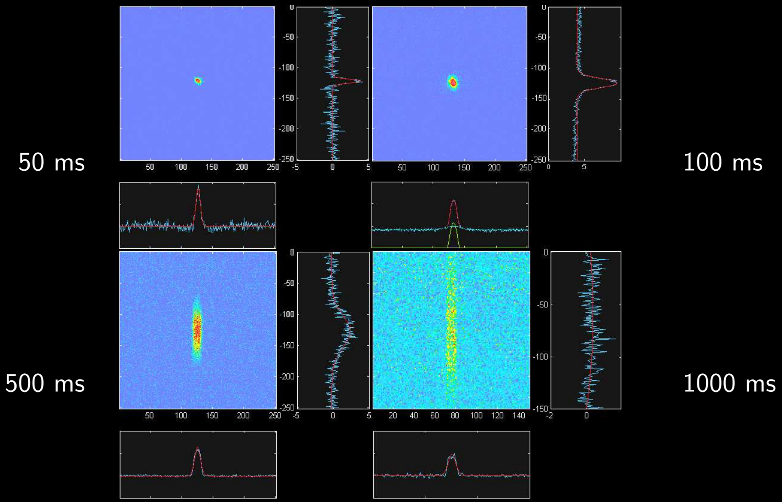
500 ms

1000 ms

10^4 atoms, 1 s free evolution time (not possible on ground)

van Zoest et al, Science 2010

BEC in microgravity – long free evolution



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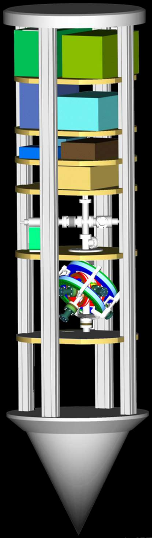
van Zoest et al, Science 2010

BEC in free fall

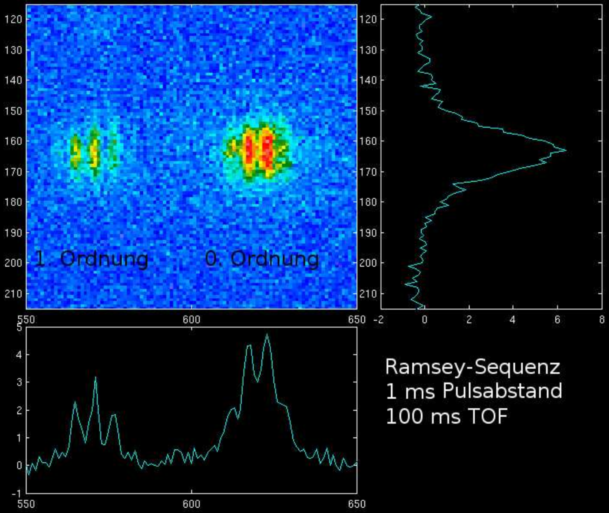
- Status
  - until now almost 200 drops
  - BEC is created regularly
  - extremely robust (survives $\sim 50\,g$)

  Worldwide most advanced technology towards space application and fundamental quantum physics in μg
- Ongoing work
  - PRIMUS (PRäzisions–Interferometrie mit Materiewellen Unter Schwerelosigkeit)
  - FOKUS (FaserOptischer FrequenzKamm Unter Schwerelosigkeit)
  - ATUS (Atom Interferometer Modeling)
  - Fluctuations in Quantum Systems
- In future
  - Fundamental Physics experiments
  - Drop tower — Texus — ISS
  - Inertial sensors
  - High precision clocks



ZARM

Interference in free fall



Ramsey-Sequenz
1 ms Pulsabstand
100 ms TOF

Outline

1. Historical remarks

Outline

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2. Newtonian orbits
   - The Kepler problem
   - The influence of the expanding universe
   - Geodesy
   - A mystery

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1. Historical remarks
2. Newtonian orbits
   - The Kepler problem
   - The influence of the expanding universe
   - Geodesy
   - A mystery
3. General Relativity
4. Orbits around a Black Hole

Outline

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Egypt

Egypt

Mexico

Mexico

Samarqand

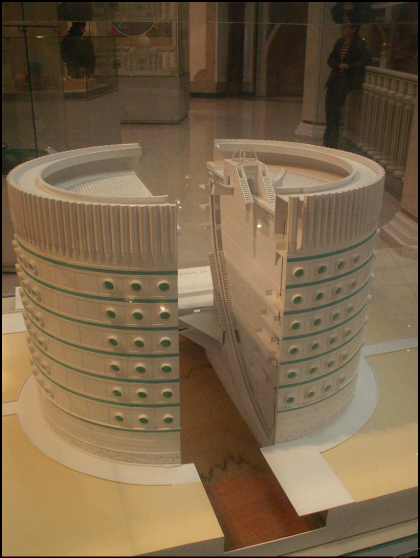
Samarqand

Ulugh Beg, 1394 — 1449

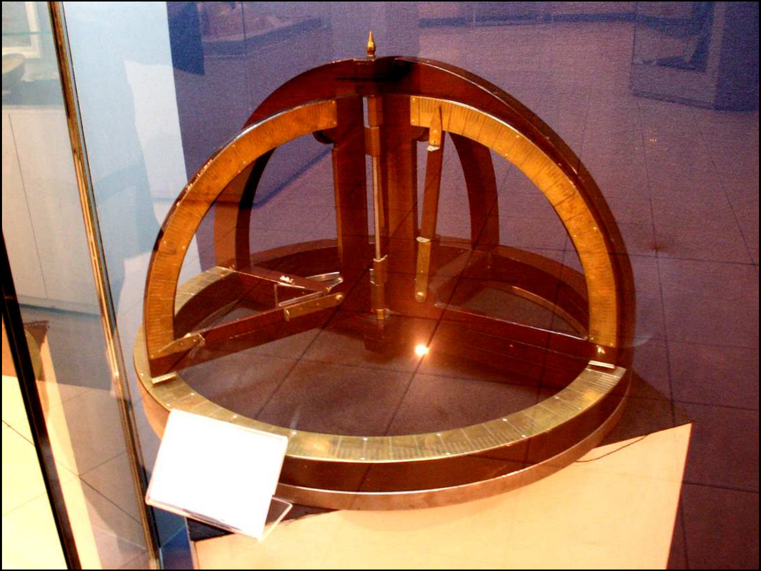
Samarqand

Samarqand

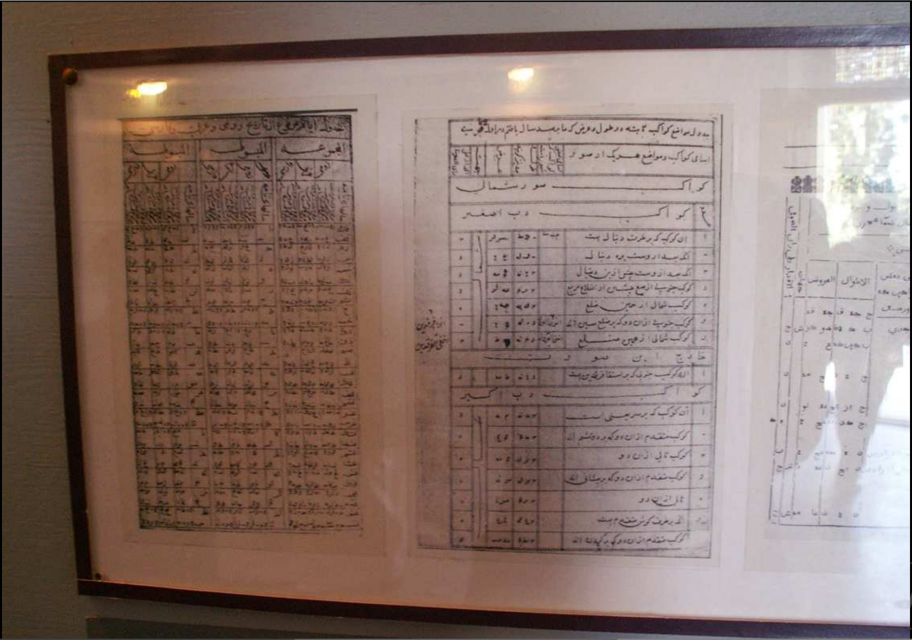
Samarqand



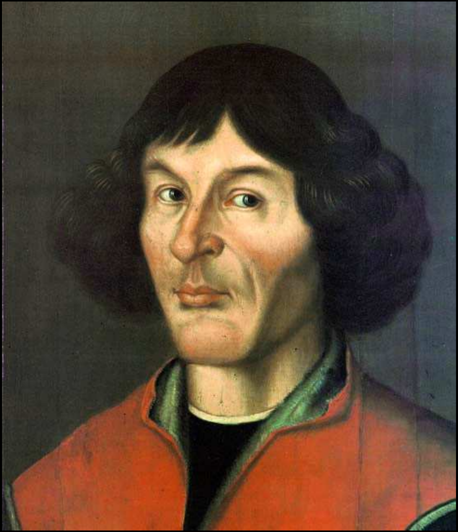
Samarqand



Samarqand



Poland



Nikolaus Kopernikus, 1473 — 1543

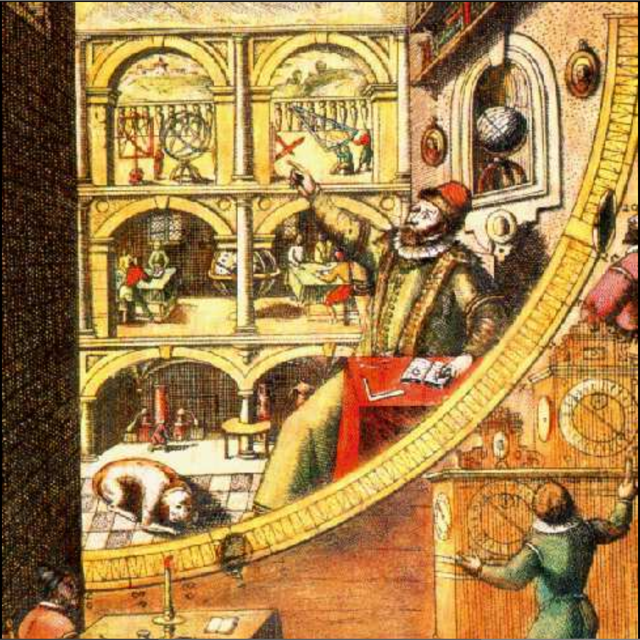
heliocentric model

Prague

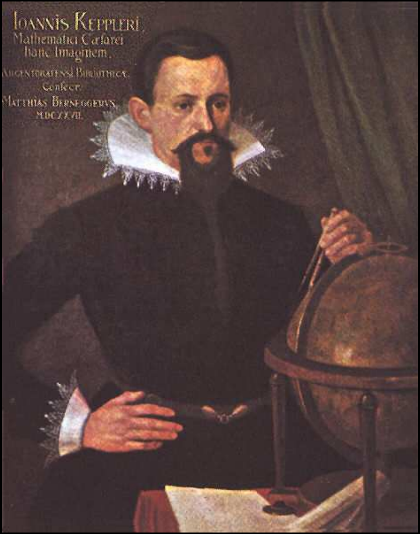
Tycho Brahe, 1546 — 1601

Precise Observations

Prague



Prague



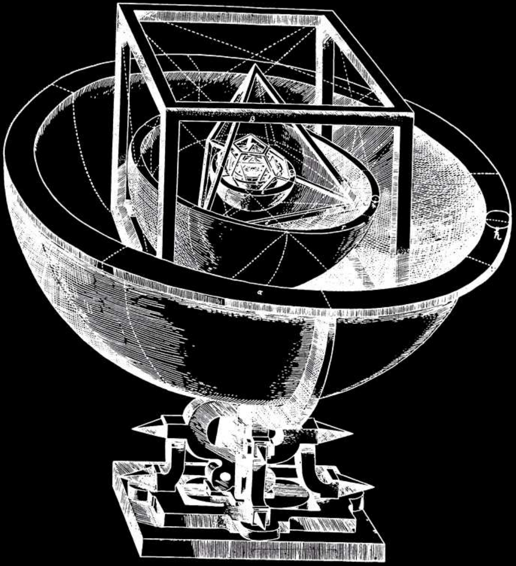
Johannes Kepler, 1571 — 1630

Kepler's laws

- 1. law (1609): the planets move on ellipses
- 2. law (1609): thw planetary orbit covers the same area in equal time spans
- 3. law (1619):

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

Prague



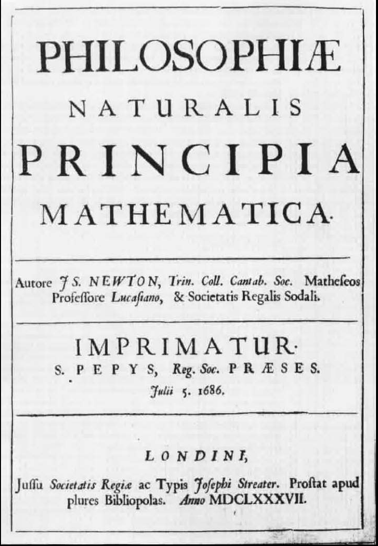
Outline

Outline

Newton

Isaac Newton
1758 — 1840

Newton


PHILOSOPHIÆ

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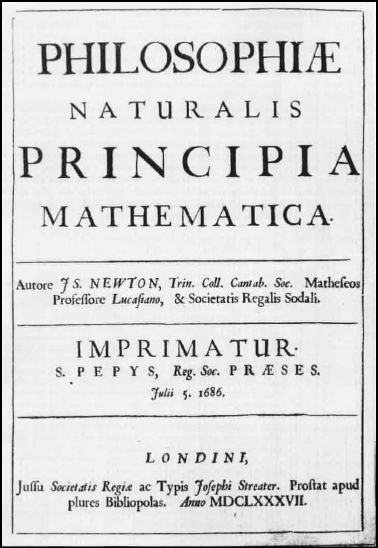
Newton axiom

$$m\ddot{\boldsymbol{r}} = \boldsymbol{F}$$

Newton



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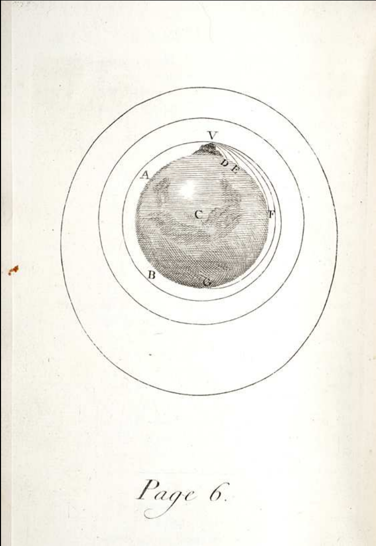
Newton's gravitational field (sphere)

$$F = -m\frac{GM}{r^2}$$

Newton



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equation of motion

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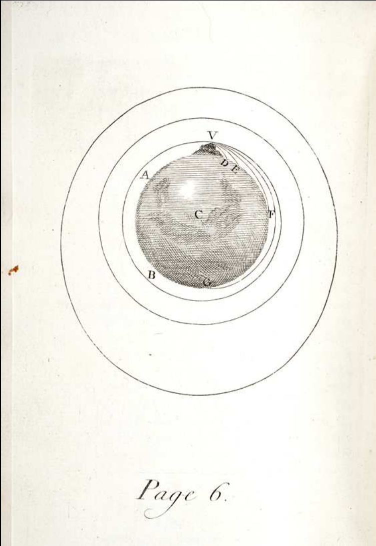

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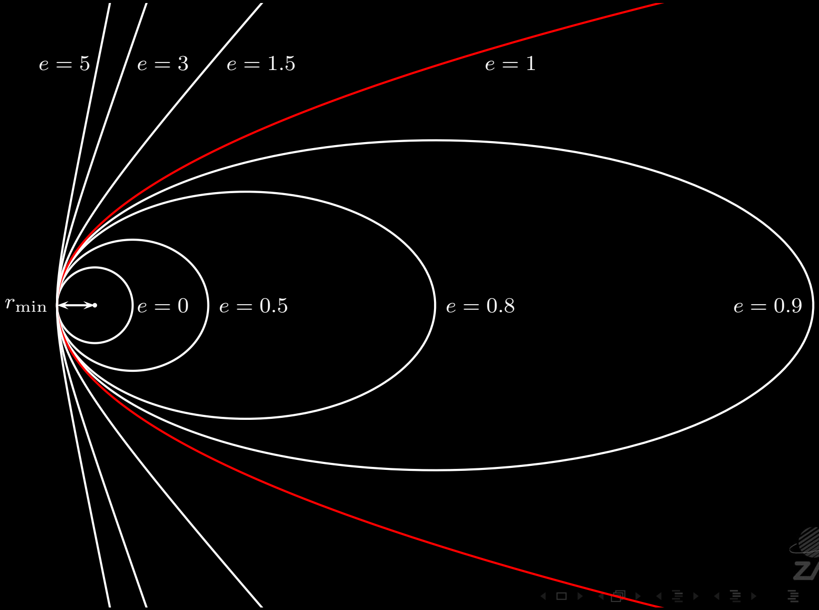
solution

$$\begin{aligned} r(\varphi) &= \frac{r_0}{1 - e\sin(\varphi - \varphi_0)} \\ t(\varphi) &= \mathcal{M}(\varphi - \sin\varphi) \end{aligned}$$

Keplerian orbits



Outline

Orbits in the expanding universe

1826: The Olbers paradox

Assumption:

- infnite universe
- homogeneity and isotropy
- static universe

Frage:

Why is the sky at night black?

Olbers paradoxon – birth of modern cosmology

- The universe has a beginning
- The universe expands
- expanding wave length
- also: finite lifetime of stars

Heinrich Wilhelm Olbers
1758 — 1840

Newtonian gravity with cosmological constant

cosmological force

$$\boldsymbol{F}_{\mathrm{cosm}} = \Lambda r$$

positive cosmological constant Λ
gives additional repulsive force

$$\ddot{r} = -\frac{GM}{r^2} + \frac{1}{3}\Lambda r$$

and slightly different orbits

- circles are slightly larger
- ellipses are slightly stretched
- hyperbolas are slightly deformed

Outline

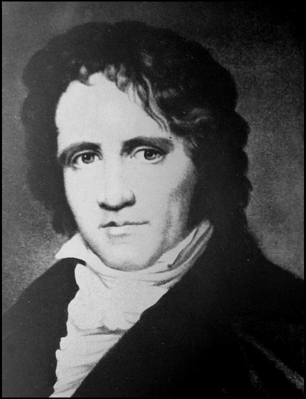
Geodesy: The shape of the Earth

Facts from Bessel's life

- worked as clerk in Bremen
- was interested in navigation and astronomy
- met Olbers who supported Bessel
- cooperation with Lilienthal astronomical observatory
- met Gauß

Bessel's scientific work

- Calculation of orbits and their disturbances
- Geodesy
  - First correct determination of flatting of Earth
  - Definition of Bessel ellipsoid
- Test of the Equivalence Principle



Friedrich Wilhelm Bessel
1784 — 1846

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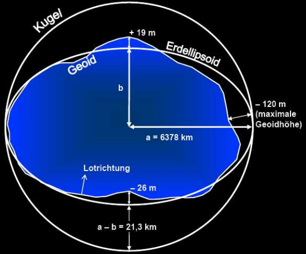

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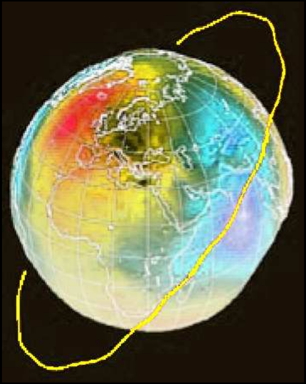

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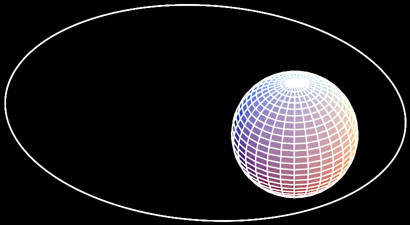
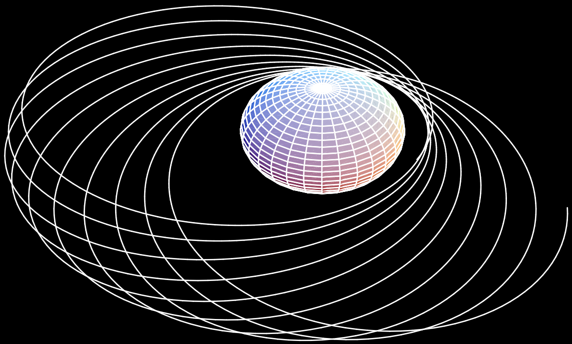
Orbits for spherical Earth or Sun



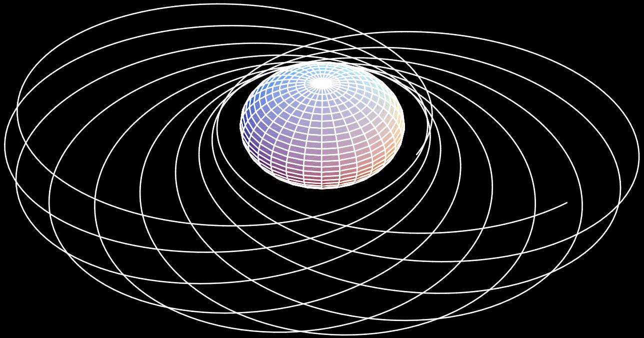
For Sphere: Ellipse or another Keplerian orbit

Orbit for flattened Earth or Sun



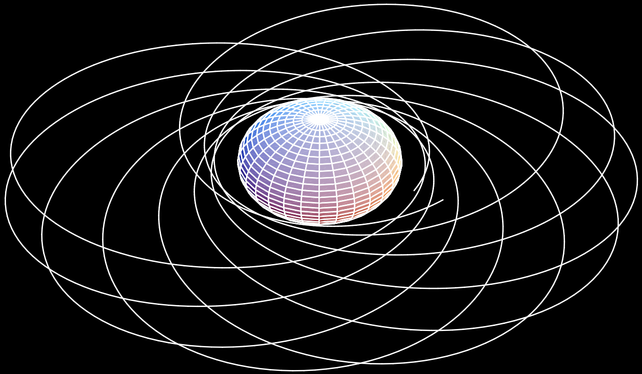
For flattened (oblate) Sun there is more mass near the equator and, thus, near Mercury → more deflection of orbit near the Sun

Orbits in the Solar system



The other planets drag Mercury outwards → Mercury stays a bit longer in the aphel position

Observed orbits



... and the really observed orbits show an even larger perihelion shift ...

Outline

Die Größe der Periheldrehung

Scientific merits:

- Calculation of orbits of Neptun from disurbances of other orbits
- Invention of weather maps

1859 observation of an anomalous perihelion shift of Mercury:

- Observational error?
- Dark matter: Planet X?
- Other multipole moments of Sun?
- Different influence of the other planets?
- Wrong theory of gravity?

Urbain Jean Joseph Le Verrier
1711 — 1877

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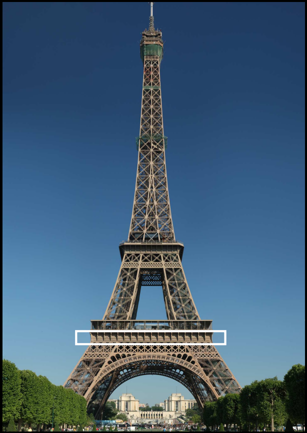


Perihelion shift

Outline

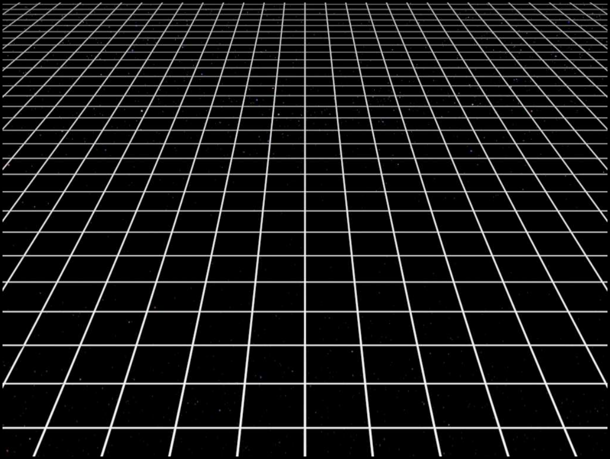
Einsteinsche Feldgleichungen

Einstein's field equations

Space–time without gravitation

The metric of flat Minkowski space–time

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

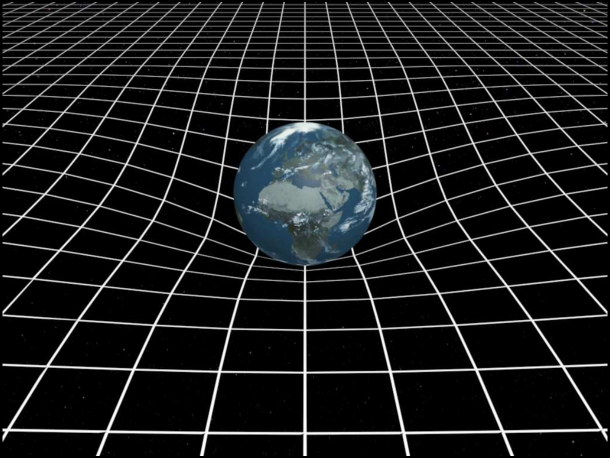


No matter in the universe: flat space–time

Space–time with gravitation

Metric of curved space–time

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

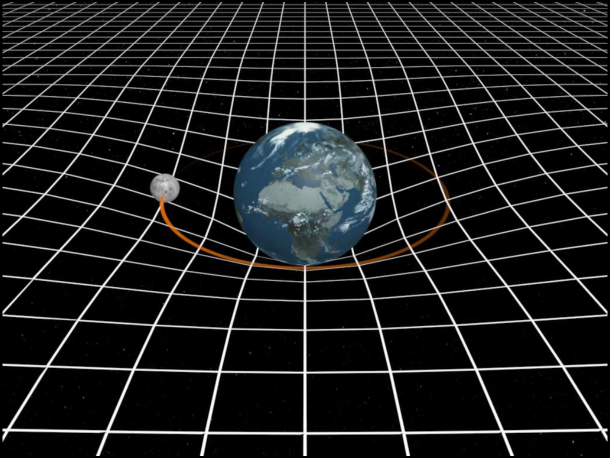


Metter determines the geometry of space and time

Motion in curved space–time

Motion of particles in curved space–time

$$0 = \frac{d^2 x^\mu}{ds^2} + \left\{ {}^{\mu}_{\rho\sigma} \right\} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds}$$



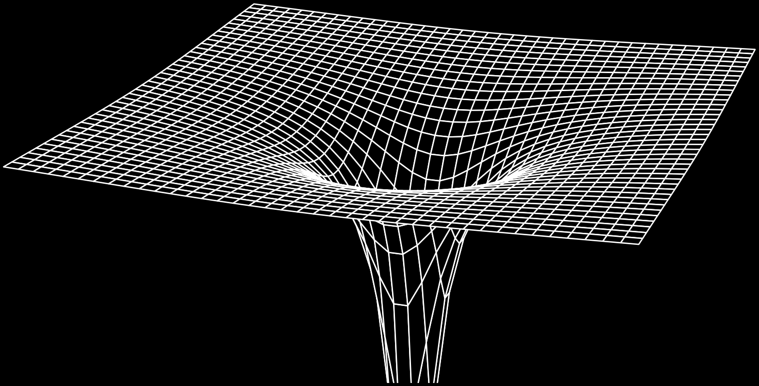
The geometry of space and time governs the motion of bodies

Strongly curves space–time

Extrapolation: strongly curved space-time: The geometry of a black hole

- Metric of curved space–time

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$



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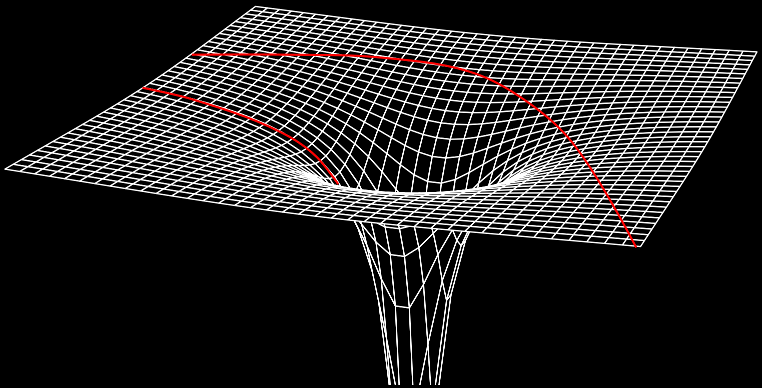
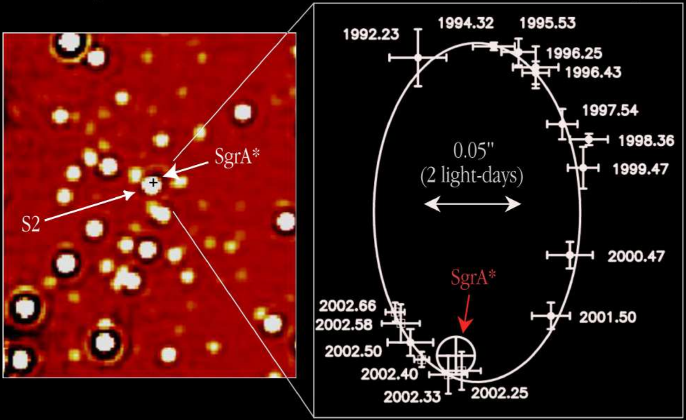


The Black Hole in the center of our Milky Way



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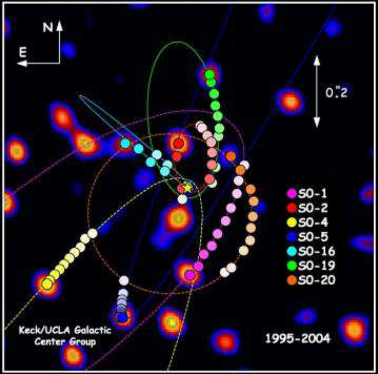


Black Hole: Mass $4 \cdot 10^6\ M_\odot$, angular velocity $\sim 1/17\ \mathrm{min}$
Only through the observation of orbits of stars one can investigate the Black Hole

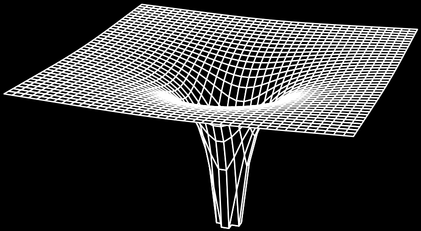
General Relativity

Einstein's field equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$$

- Nonlinear system of partial differential equation for $g_{\mu\nu}$
- Known solutions:
  - Schwarzschild, Schwarzschild–de Sitter
  - Reissner–Nordström, Reissner–Nordström–de Sitter
  - Kerr
  - Kerr–Newman, Kerr–Newman-de Sitter
  - Taub–NUT
  - Plebański–Demiański
  - Staubscheibe
  - Solutions with strings

Gravitation
$=$ Deformation of space–time
$\widehat{=}$ Deformation of elastic bodies



General Relativity

Einstein's field equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

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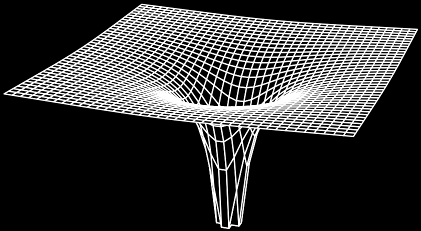


Not only esoteric theory: Applications

Scientific applications

- High precision tests of General Relativity
  - Perihelion shift
  - Red shift
  - gravitational time delay
  - Light deflection, fybys of satellites
  - Gravitomagnetism, Lense–Thirring–effect (LAGEOS)
- Binary systems (astrophysics)
- Satellite dynamics (flyby–anomaly, Pioneer–anomaly)
- Gravitational waves

Practical Applications

- Clocks in space (ACES, OPTIS, SPACETIME, EGE, SAGAS)
- Precise navigation and positioning (GPS, Galileo)
- Geophysivs (measuring the gravitational field, tectonics, continental drift, climate research, ocean warming, ...)
- Definition of international atomic time TAI
- Definition of reference systems (for astrophysics, satellite dynamics, ...)

Outline

The horizon – Schwarzschild–radius

- Escape velocity from surface of Earth

$$\begin{aligned}\text{kinetic energy} &= \text{potential energy} \\ \frac{1}{2}mv^2 &= m\frac{GM}{r}\end{aligned}$$

r smaller $\rightarrow$ v larger

- escape velocity

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}}$$

For Earth approx. 11,2 km/s.

The horizon – Schwarzschild–radius

- Escape velocity from surface of Earth

$$\begin{aligned} \text{kinetic energy} &= \text{potential energy} \\ \frac{1}{2}mv^2 &= m\frac{GM}{r} \end{aligned}$$

  r smaller $\rightarrow$ v larger

- escape velocity

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}}$$

  For Earth approx. 11,2 km/s.
  For Hayabusa approx. 2 cm/s.

The horizon – Schwarzschild–radius

- maximal velocity = velocity of light c $\leftrightarrow$ smallest r
- gilt bei

$$\frac{1}{2} m c^2 = m \frac{GM}{r_{\min}} \qquad \Rightarrow \qquad r_{\min} = r_{\rm S} = \frac{2GM}{c^2}$$

- therefore there is a smallest distance for leaving the region of gravitational influence of a mass M
- for smaller r there is no escape

there is a horizon $\Leftrightarrow$ Black Hole

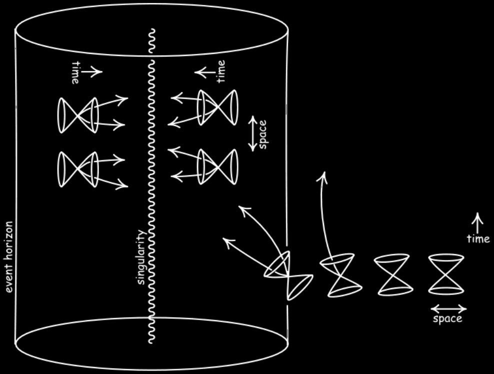
- it is possible to enter the hole but not to leave it: semitransparent membrane
- M within a sphere of radius $R_{\rm S}$ gives minimal density $\rho = \dfrac{3c^2}{8\pi G} \dfrac{1}{R_{\rm S}^2}$

  the larger a Black Hole is, the smaller is its matter density

Schwarzschild space–time

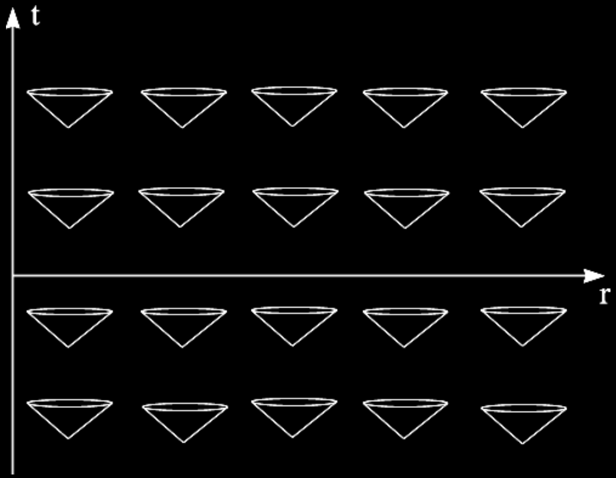
Metric (Schwarzschild 1916)

$$ds^2 = \left(1 - \frac{R_\mathrm{S}}{r}\right) dt^2 - \frac{1}{1 - \frac{R_\mathrm{S}}{r}} dr^2 - r^2 d\vartheta^2 - r^2 \sin^2 \vartheta d\varphi^2$$



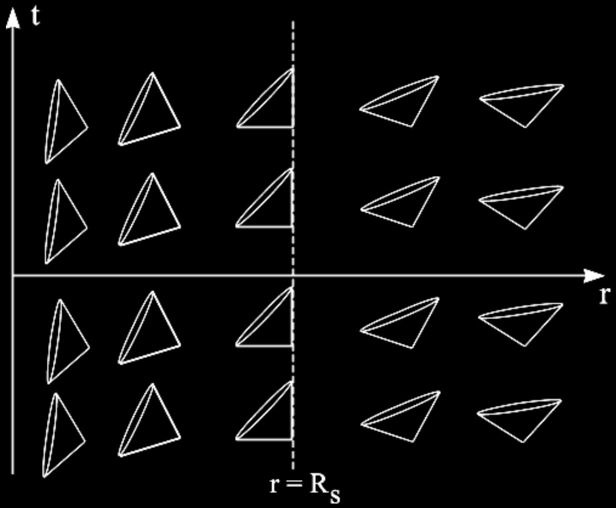
Karl Schwarzschild
1873 — 1916

Schwarzschild space–time: Light cones



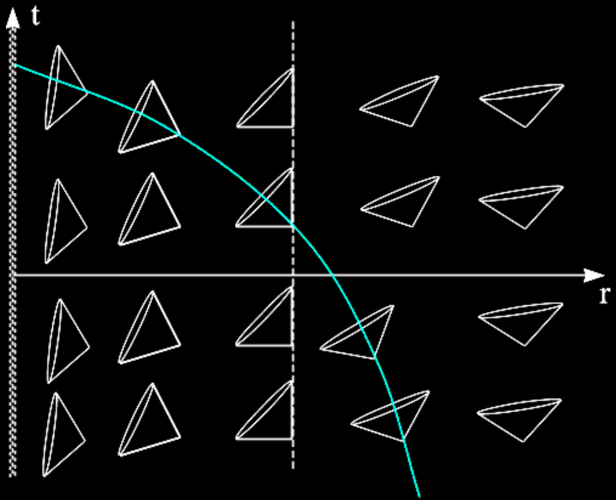
Light cones in Minkowksi space–time

Schwarzschild space–time: Light cones



Light cones in Schwarzschild space–time

Schwarzschild space–time: Light cones



Orbit of massive particle in Schwarzschild space–time

Orbits around a Schwarzschild Black Hole

Old equation of motion

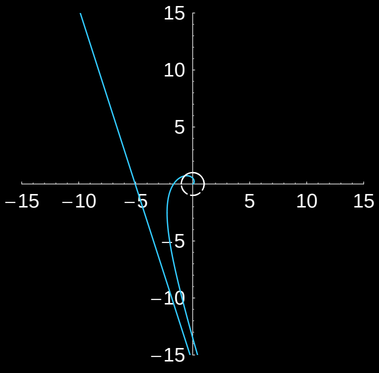
$$\ddot{r} = -\frac{M}{r^2} + \frac{L^2}{r^3}$$

New equation of motion

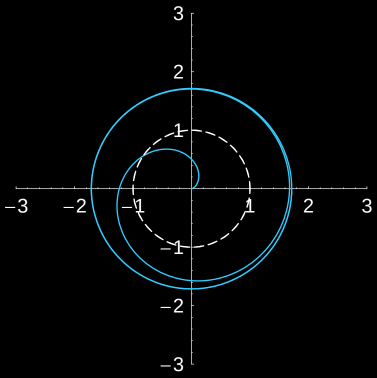
$$\ddot{r} = -\frac{M}{r^2} + \frac{L^2}{r^3} - 3\frac{ML^2}{r^4}$$

Gives 26 types of orbits (Newton: 4 types of orbits)

Orbits around a Schwarzschild Black Hole

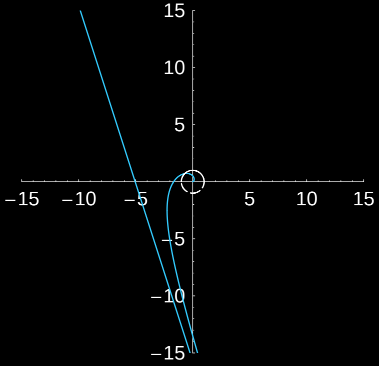


terminating orbit

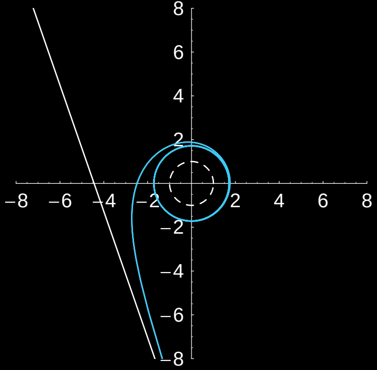


hyperbolic spirals

Orbits around a Schwarzschild Black Hole

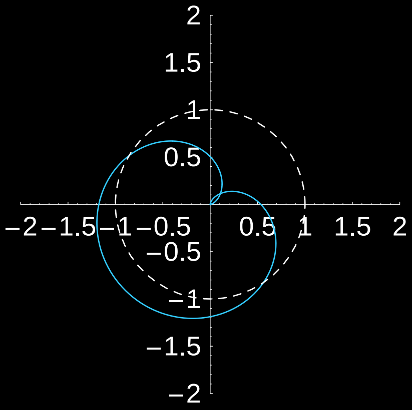


terminating orbit

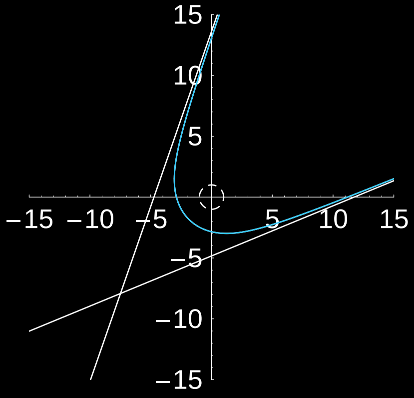


hyperbolic spirals

Orbits around a Schwarzschild Black Hole

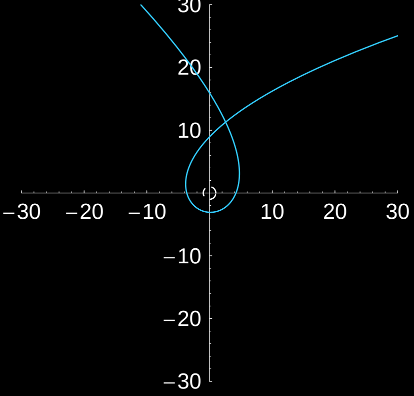


pseudo–hyperbolic

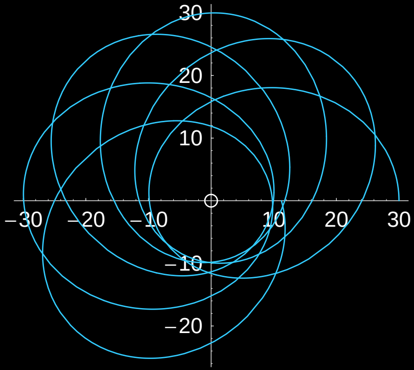


quasi hyperbolic

Orbits around a Schwarzschild Black Hole

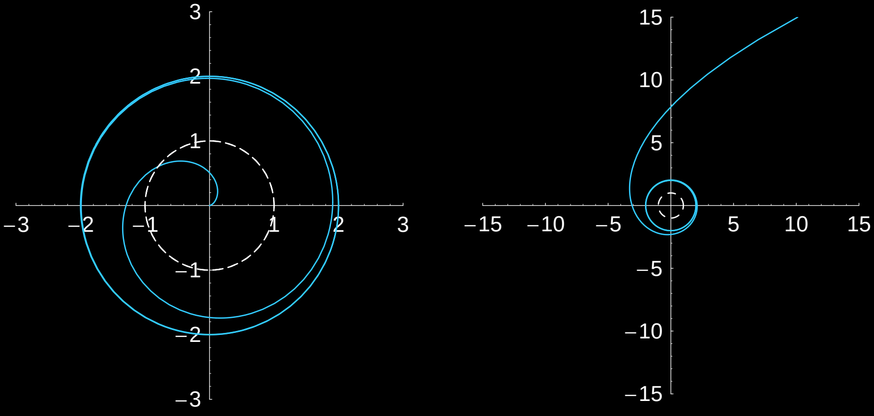


quasi parabolic



quasi–elliptic: Perihelion shift

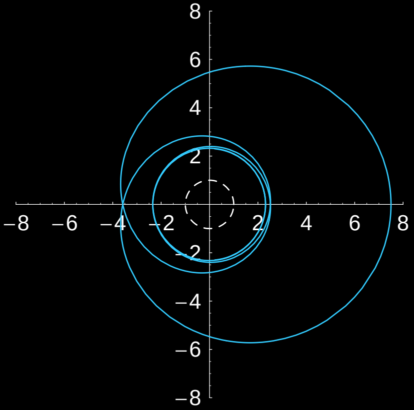
Orbits around a Schwarzschild Black Hole



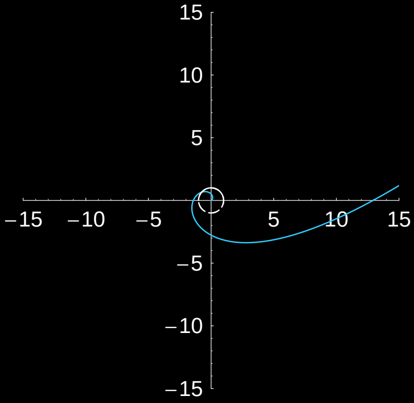
finite parabolic spiral

infinite parabolic spiral

Orbits around a Schwarzschild Black Hole

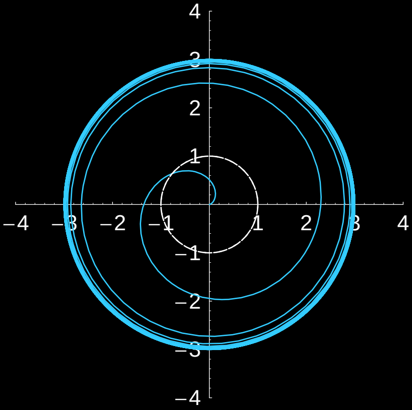


double spiral (Poincaré)

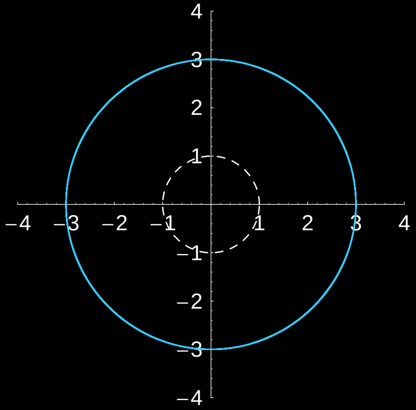


pseudo–parabolic

Orbits around a Schwarzschild Black Hole

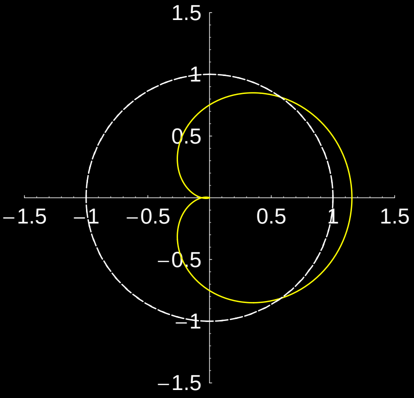


spiral

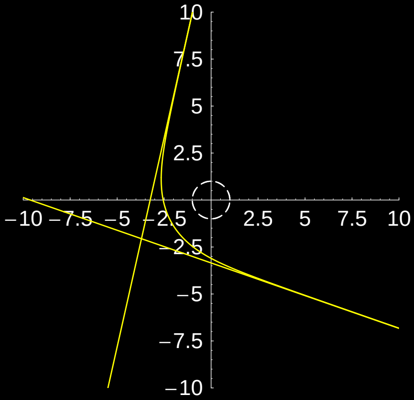


circle (ISCO)

Orbits of light around a Schwarzschild Black Hole

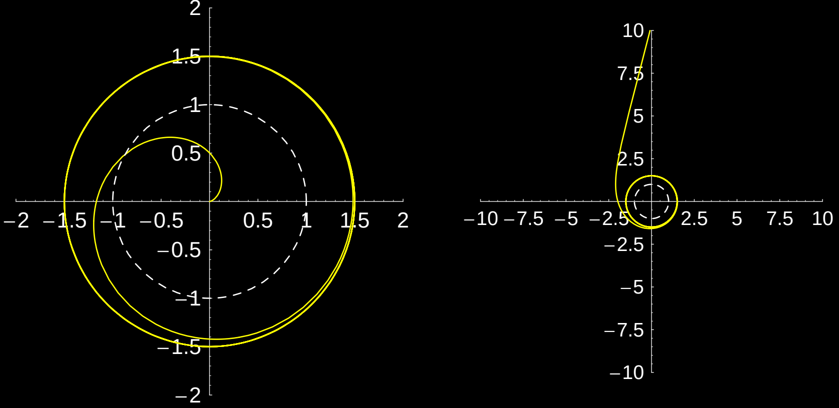


pseudo–hyperbolic



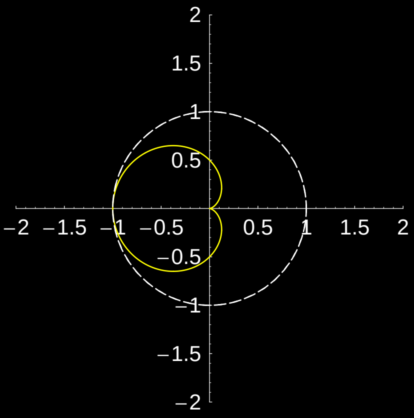
quasi–hyperbolic

Orbits of light around a Schwarzschild Black Hole

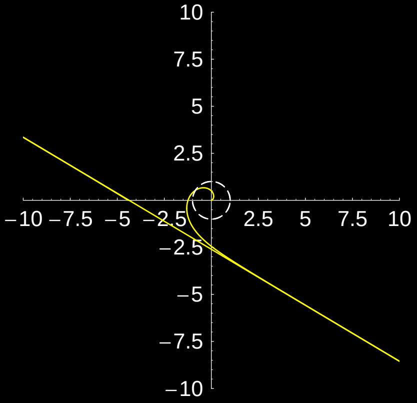


finite pseudo–hyperbolic spiral — infinie quasi–hyperbolic spiral — ISCO

Orbits of light around a Schwarzschild Black Hole

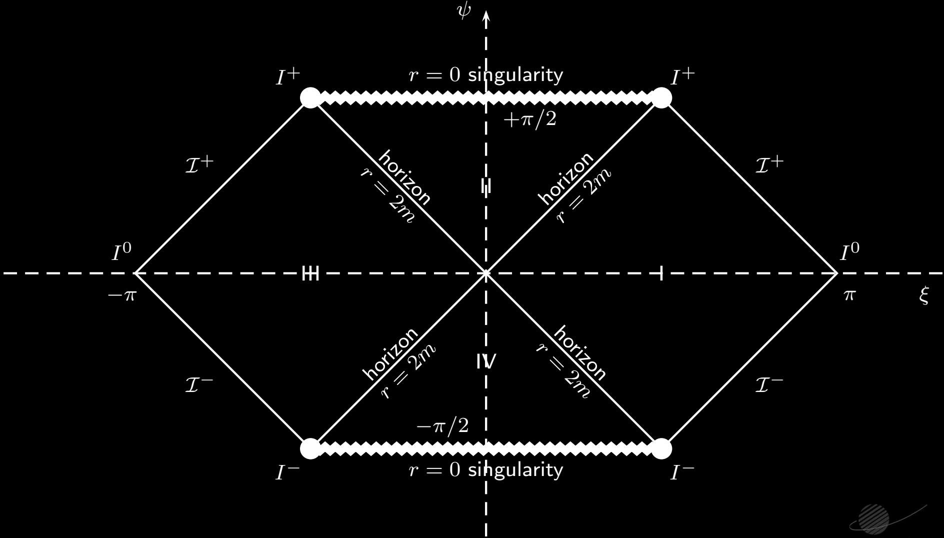


spiral



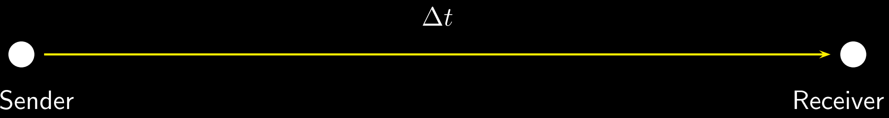
pseudo–hyperbolic

Schwarzschild space–time



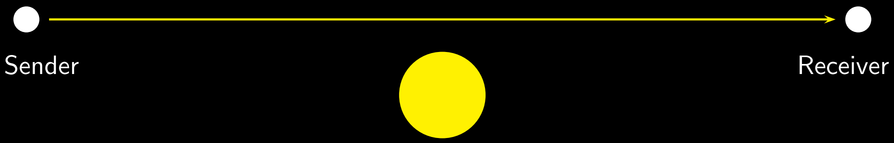
Schwarzschild space–time in Carter–Penrose diagram

Gravitational time delay

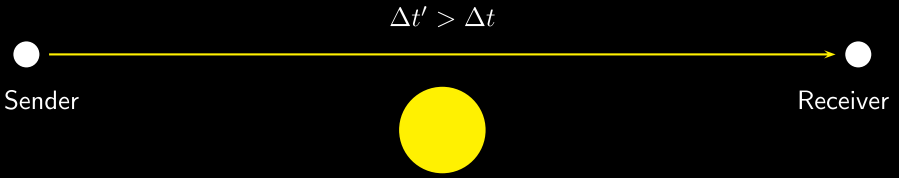


Gravitational time delay

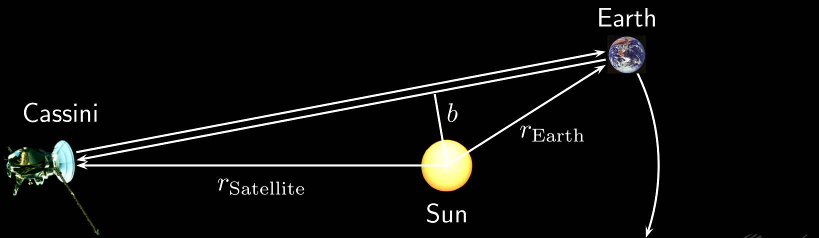
$$\Delta t' > \Delta t$$



Gravitational time delay



Cassini–Experiment



Gravitational redshift

Outline

Orbits around charged Black Holes

Reissner–Nordström space–time

$$ds^2 = \left(1 - \frac{M}{r} + \frac{q^2}{r^2}\right) dt^2 - \frac{1}{1 - \frac{M}{r} + \frac{q^2}{r^2}} dr^2 - r^2 d\vartheta - r^2 \sin^2 \vartheta d\varphi^2$$

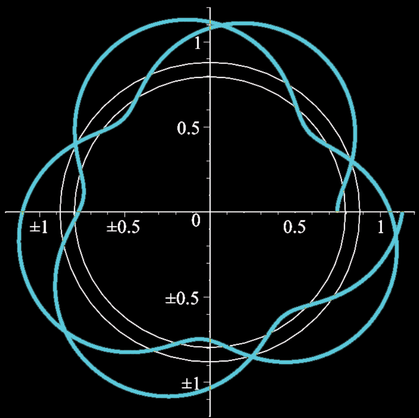
Old equation of motion

$$\begin{aligned}\ddot{r} &= -\frac{M}{r^2} + \frac{L^2}{r^3} \\ \ddot{r} &= -\frac{M}{r^2} + \frac{L^2}{r^3} - 3\frac{ML^2}{r^4}\end{aligned}$$

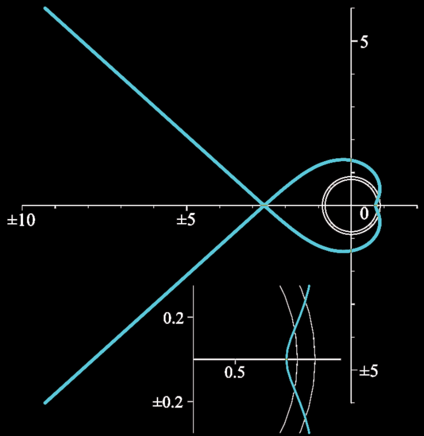
New equation of motion

$$\ddot{r} = -\frac{M}{r^2} + \frac{L^2}{r^3} - 3\frac{ML^2}{r^4} + \frac{q^2}{r^3} + 2\frac{q^2L^2}{r^5}$$

Orbits around charged Black Holes

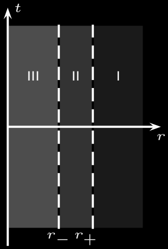


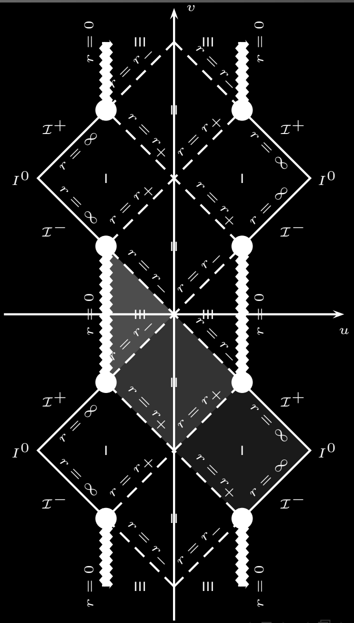
Periodic transition into an infinite series of new universes



Escape orbit: Escape into a different universe

Reissner-Nordström space–time





Outline

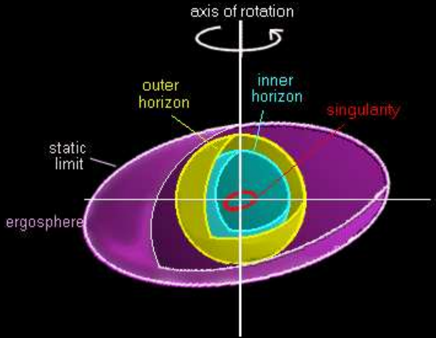
Rotating Black Holes

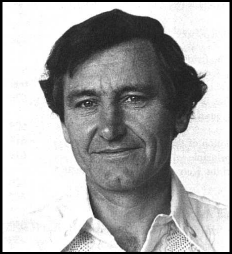
Kerr space–time

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\sin^2\theta}{\rho^2}\left(adt - \rho_0^2 d\phi\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

mit $\rho^2 = r^2 + a^2\cos^2\theta$, $\Delta = r^2 - 2Mr + a^2$

a angular momentum of Black Hole

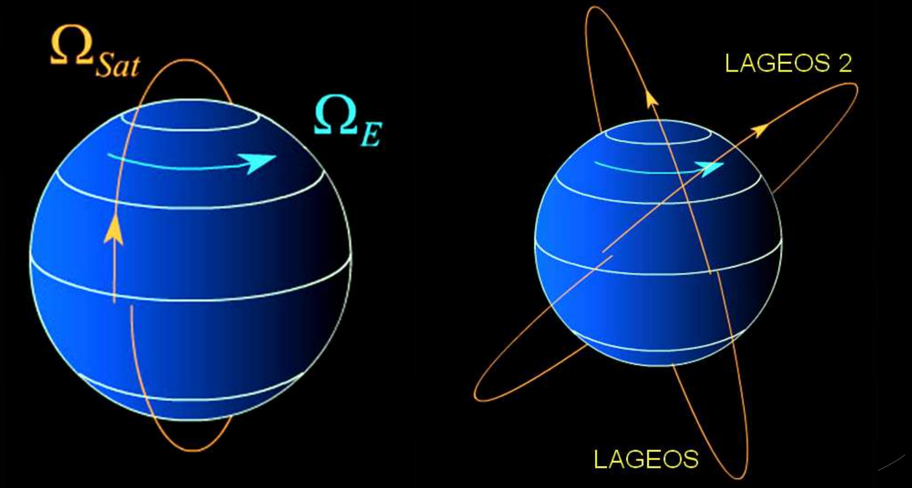




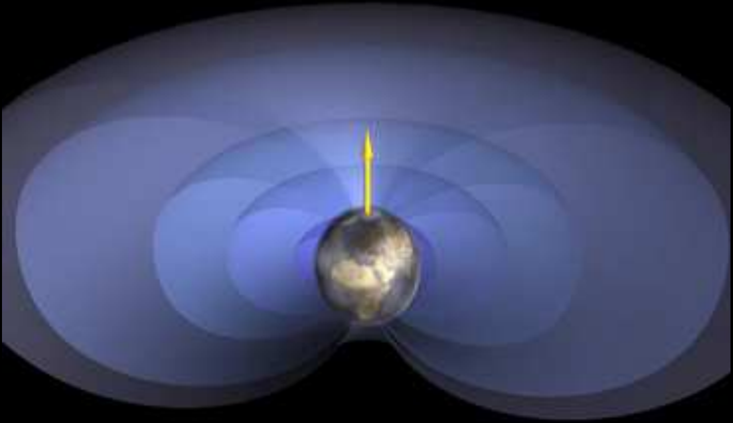
Roy Kerr *1934

Gravitomagnetism

Orbits leave the initial orbital plane

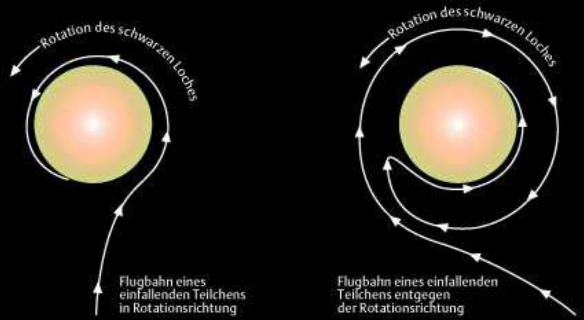


Gravitomagnetism

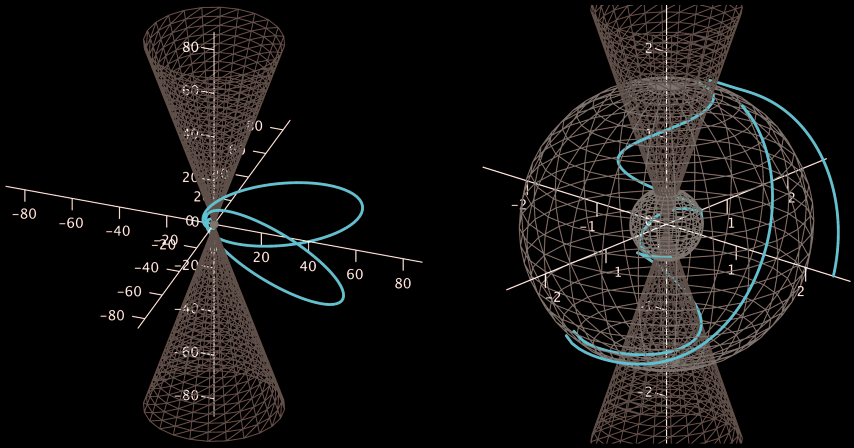


Lense–Thirring–effect

Orbits in Kerr space–time

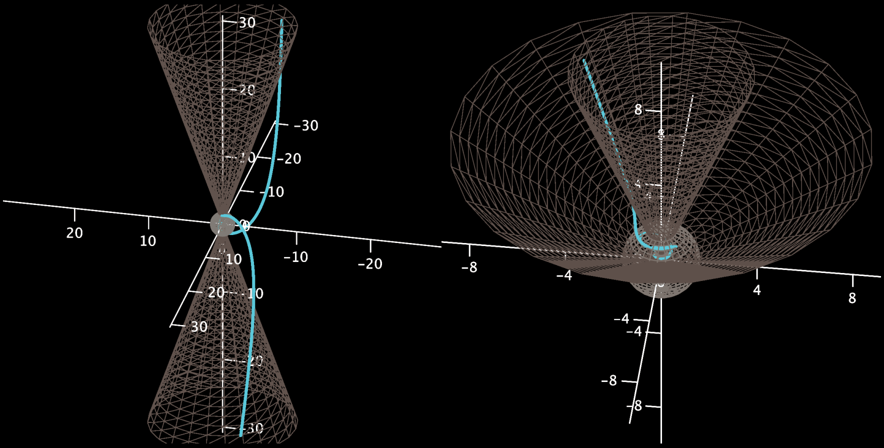


Orbits in Kerr space–time



bound orbits

Orbits in Kerr space–time



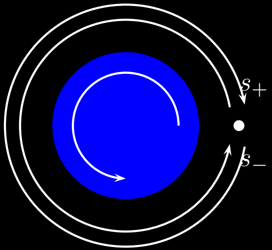
escape orbits

Gravitomagnetic clock effect

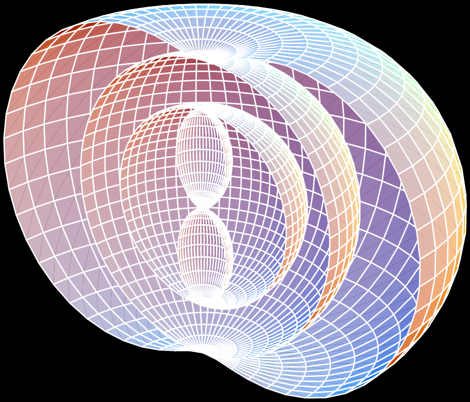
Time difference between two counter–rotating clocks

$$s_+ - s_- = 4\pi \frac{J}{M} \sim 10^{-7}\ \mathrm{s}$$

is topological effect



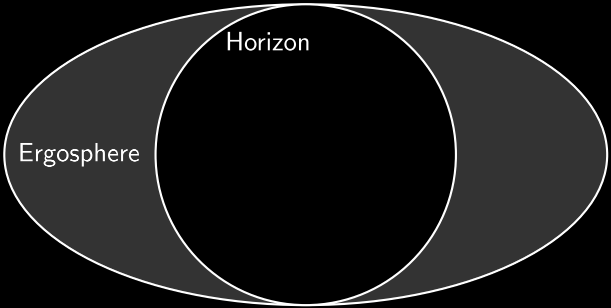
Ergosphere



Horizon and ergosphere in Kerr space–time

Ergosphere

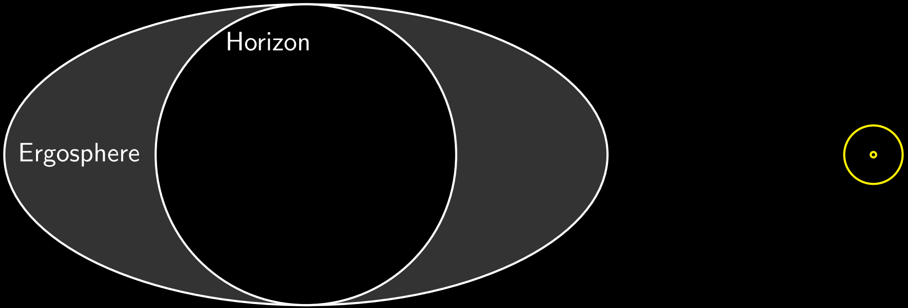
Light propagation in the vicinity of a rotating Black Hole



Horizon and ergosphere in Kerr space–time

Ergosphere

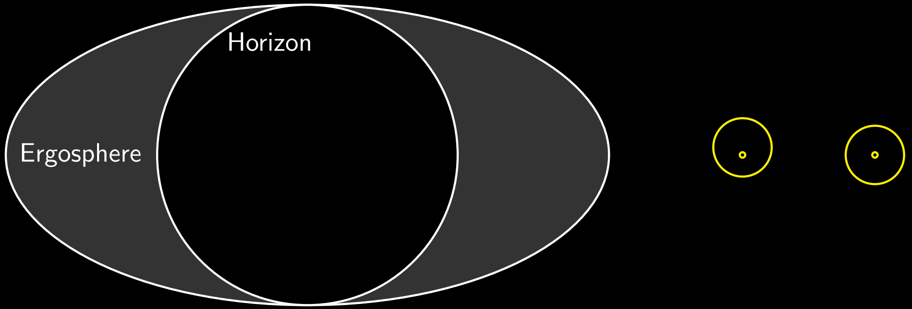
Light propagation in the vicinity of a rotating Black Hole



Horizon and ergosphere in Kerr space–time

Ergosphere

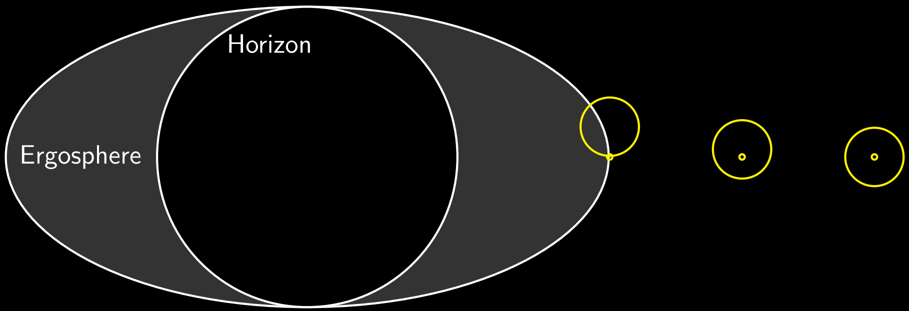
Light propagation in the vicinity of a rotating Black Hole



Horizon and ergosphere in Kerr space–time

Ergosphere

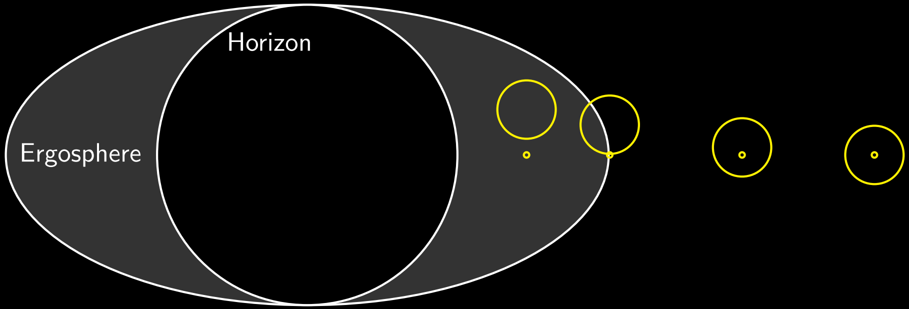
Light propagation in the vicinity of a rotating Black Hole



Horizon and ergosphere in Kerr space–time

Ergosphere

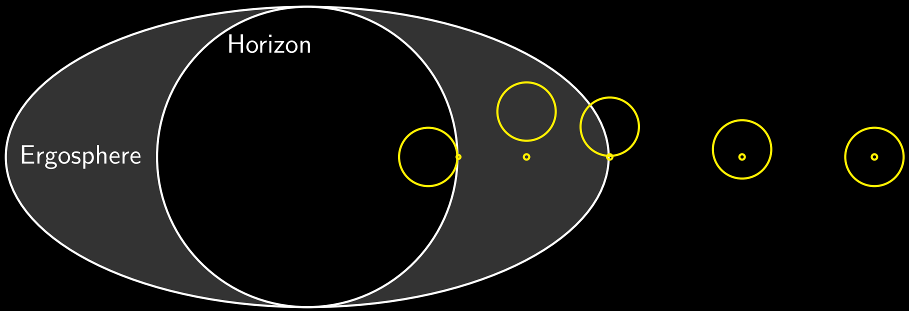
Light propagation in the vicinity of a rotating Black Hole



Horizon and ergosphere in Kerr space–time

Ergosphere

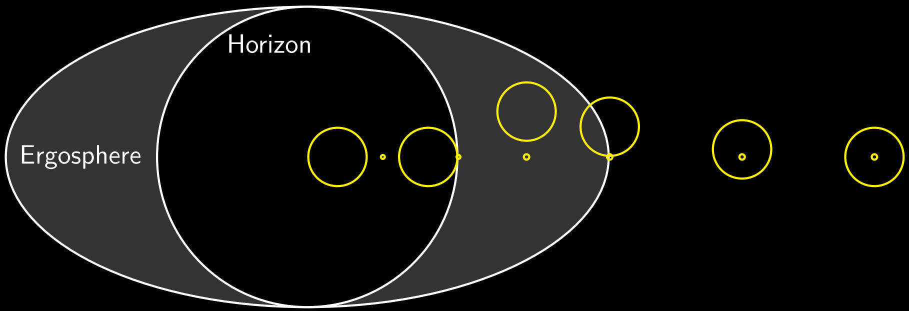
Light propagation in the vicinity of a rotating Black Hole



Horizon and ergosphere in Kerr space–time

Ergosphere

Light propagation in the vicinity of a rotating Black Hole



Horizon and ergosphere in Kerr space–time

Outline

Taub–NUT space–times

Metric

$$ds^2 = \frac{\Delta}{\rho^2}\left(dt - 2n\cos\vartheta d\varphi\right)^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right)$$

with

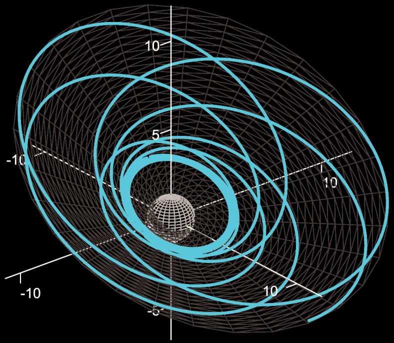
$$\rho^2 = r^2 + n^2\,, \qquad \Delta = r^2 - 2Mr - n^2$$

$n =$ magnetic masses

- There are two horizons: one for positive r, the other at negative r
- Orbits in equatorial plane: For smaller radius the circumference remains finite

Orbits in Taub–NUT space–times

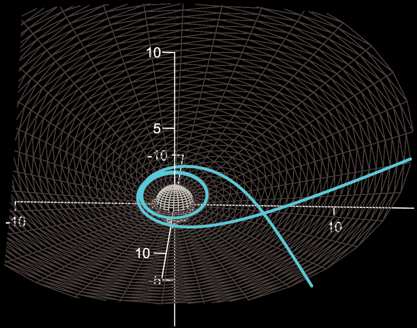
- Orbits lie on a cone
- Orbits may reach negative r
- Perihelion shift
- No Lense–Thirring effect



bound (planetary) orbit

Orbits in Taub–NUT space–times

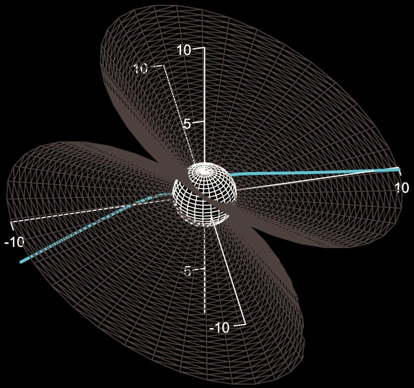
- Orbits lie on a cone
- Orbits may reach negative r
- Perihelion shift
- No Lense–Thirring effect



escape orbit

Orbits in Taub–NUT space–times

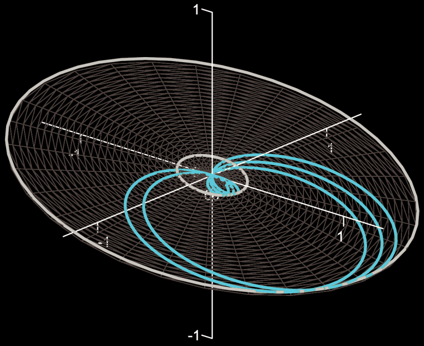
- Orbits lie on a cone
- Orbits may reach negative r
- Perihelion shift
- No Lense–Thirring effect



crossover transit–orbit

Orbits in Taub–NUT space–times

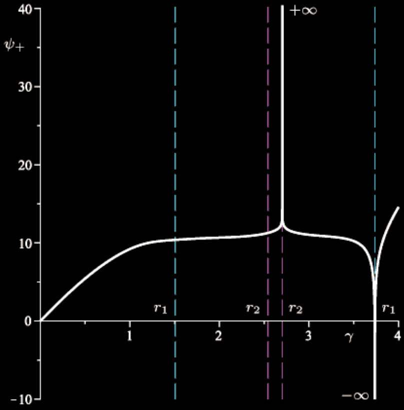
- Orbits lie on a cone
- Orbits may reach negative r
- Perihelion shift
- No Lense–Thirring effect



crossover orbit (truning points are on the horizon)

Taub–NUT space–times: incompleteness

- In Taub–NUT space–time there is no curvature singularity
- However, is geodetic incomplete ...
  At second transit through horizon the proper time remains finite while one moves through the whole future of the universe (increasing fast motion effect)

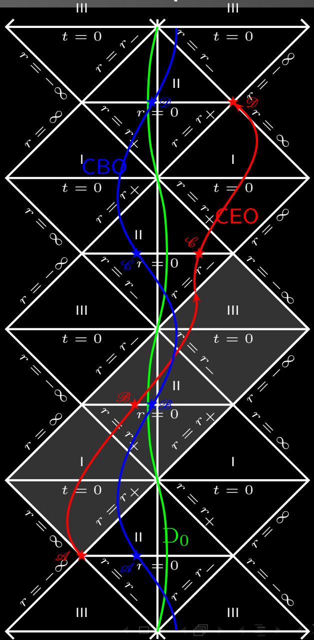


Orbits in Taub–NUT space-times

geodetic Incompleteness

Taub–NUT space–times: geodetic incompleteness

Taub–NUT with and without analytic continuation



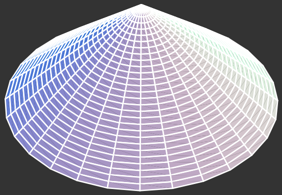
Outline

Schwarzschild with string

Metric

conical Schwarzschild space–time

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 \left(d\vartheta^2 + \beta^2 \sin^2 \vartheta d\varphi^2\right)$$

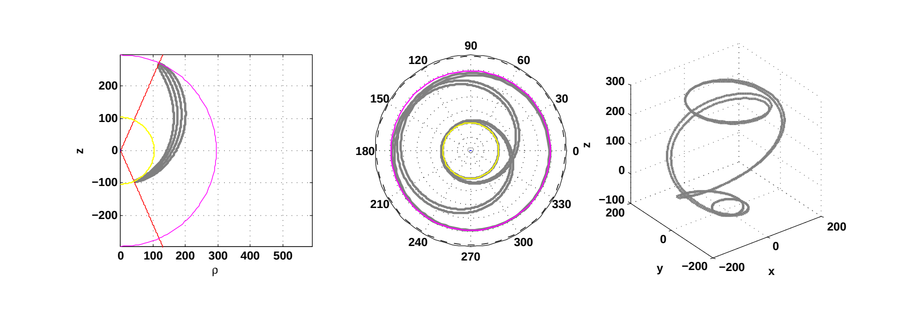


additional material string along z–axis

- geodesic equation looks similar to Schwarzschild
- φ–motion modified by β
- leads to additional perihelion shift, light deflection (implications for possible observations)

Schwarzschild with string

- Motion does not remain in equatorial plane



bound orbit – Poincaré's double circle

Schwarzschild with string

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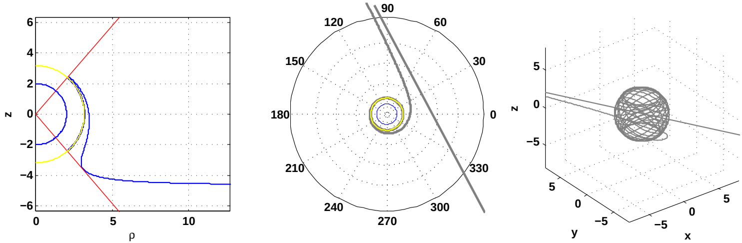

spiral escape orbit — tries to cover whole sphere

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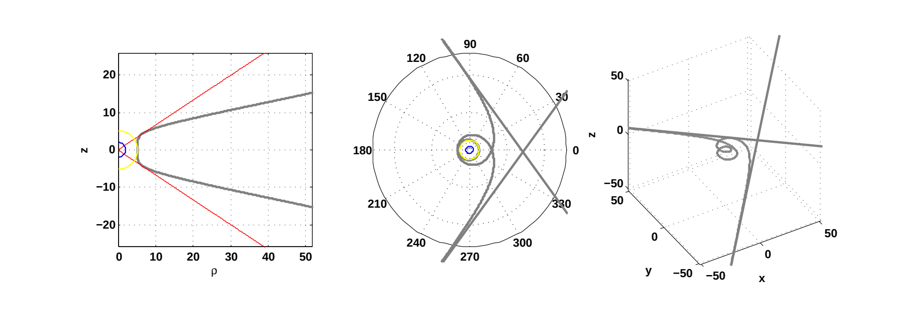

escape orbit

Kerr with string

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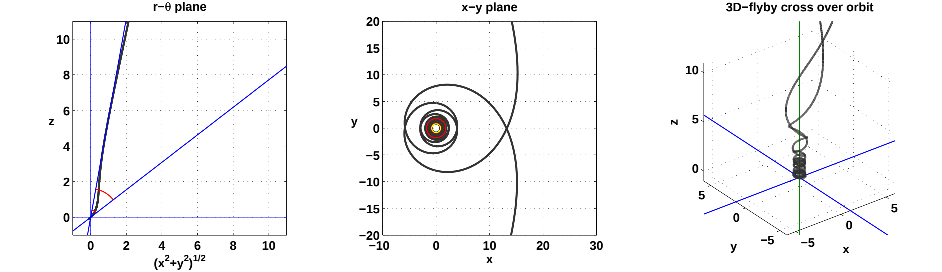

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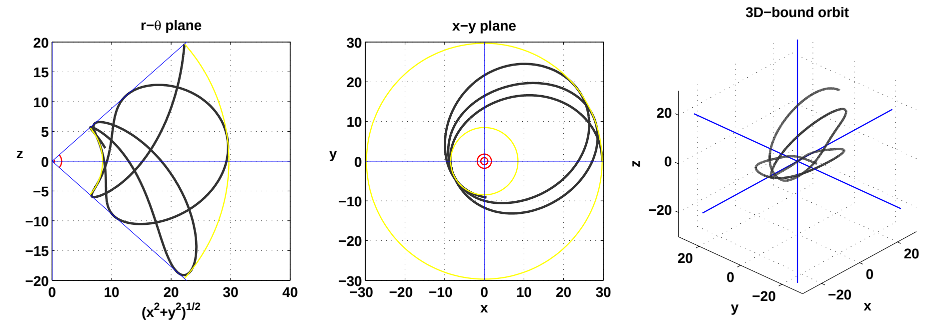

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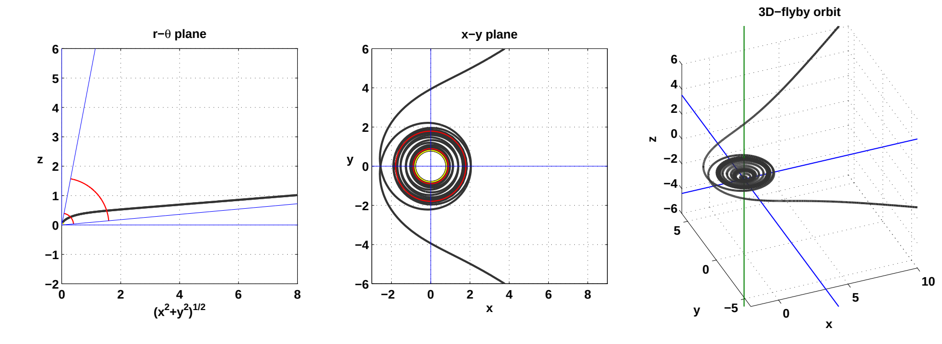
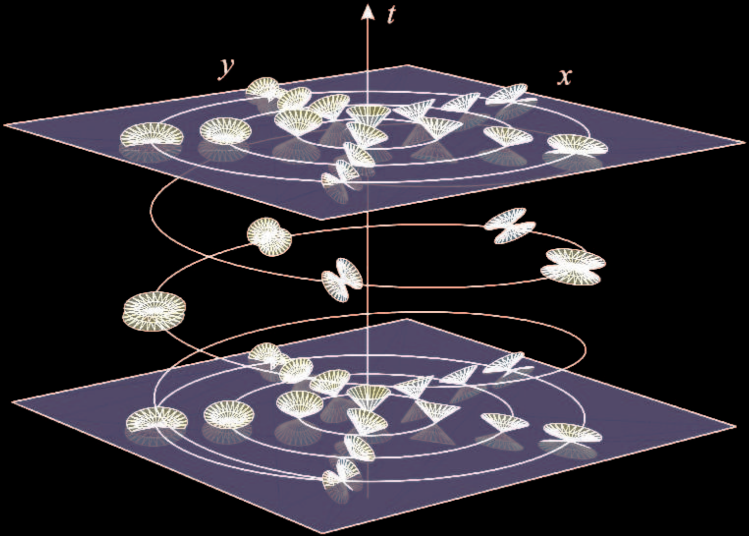


escape orbit

Outline

Gödel cosmos



Outline

Summary – Outlook

Summary

- Orbits have to be treated within the frame of General Relativity
- Within General Relativity ther are much more orbits than in Newtonian gravity
- There appear strange effects
- There are singularities ($\leftrightarrow$ is a serious problem — may be solved within quantum gravity)

Not covered

- Wormholes
- warp drive
- Particles with spin
- Motion of quantum particles

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Bisher unerklärte Beobachtungen

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Gravitation bei großen Abständen? Schwache Gravitationsfelder?

Acknowledgement

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- Hanns Selig (train station)

und

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- University Bremen

More information

Thank you for your attention!

http://www.zarm.uni-bremen.de
→ space science
→ fundamental physics